**Hotelling line**

The Hotelling line describes how products are differentiated.

The simplest version of the model is the case in which two producers and differentiate products in only one dimension. In the simple model we assume that consumers are uniformly distributed in the interval .

Let represent the position of producer , represent the position of producer and represent the position of the consumer who is indifferent between and .

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The utility function of each consumer is given by the following formula where represents the private valuation of the consumer, represents the price of product and represents the transport cost.

The indifferent consumer is always located between and so the absolute differences in the utility function can be rewritten.

The position of the indifferent consumer is calculated using the definition of indifference. The utilities of the indifferent consumer at position must be equal.

The utility functions are continuous with respect to so the demands for each producer depend only on the position of the indifferent consumer.

The profits of firm A are equal to revenues.

The firm maximizes profits by setting prices. The profit-maximizing prices are obtained by settings the first-order condition to zero.

Similarly, the profits of firm are equal to revenues and the firm sets prices to maximize profit.

The expressions derived above are the best response functions of the producers. They form a system of two linear equations with two variables and . The equilibrium prices and solve the system.

The position of the indifferent consumer in equilibrium is given by the following expression.

Equilibrium profits are given by the following expressions.

If it is assumed that , then, if firm chooses price , firm has two options. The first option is to choose price and share the market equally. The second option is to choose price and capture the whole market.

Differentiation increases profits because it allows firms to raise prices but only if the firms are sufficiently differentiated.

The decision to undercut the other firm results in the following payoff matrix.

|  |  |  |
| --- | --- | --- |
| Firm \ Firm | Share | Capture |
| Share | , | , |
| Capture | , | , |

The firms can employ a grim trigger strategy to discourage undercutting. This strategy maximizes profit in the long-run if is low enough.

The grim trigger strategy results in more space to raise prices and therefore decreases the utility of differentiation compared to the single-period decision. If the discount rate is high/low then the firms will tend to differentiate more/less.